1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part 1, Class XI)
Module Name/Title	Unit 3, Module 6, Circular motion and Banking of Roads Chapter 5, Laws of motion
Module Id	Keph_10506_eContent
Pre-requisites	Concept of friction and circular motion, Resolution of vectors
Objectives	After going through this lesson, the learners will be able to • Deduce the meaning and expression of centripetal force • Recognise the need for centripetal force for any object to travel in a curved path • Understand the Significance of banking of roads.
Keywords	Centripetal force, Banking of roads

2. Development Team

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1. UNIT SYLLABUS

Chapter 5: Laws of Motion

Intuitive concept of force, Inertia, Newton's first law of motion, momentum and Newton's second law of motion, Impulse; Newton's third law of motion.

Law of conservation of linear momentum and its applications.

Equilibrium of concurrent forces, Static and kinetic friction, laws of friction, rolling friction, lubrication.

Dynamics of uniform circular motion: Centripetal force, examples of circular motion (vehicle on a level circular road, vehicle on banked road).

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 7 Modules

The above unit is divided into seven modules as follows:

Module 1	• Force • inertia • First law of motion
Module 2	• Momentum • Second law • Impulse

	• F=ma • Constant and variable force
Module 3	• Third law • Conservation of linear momentum and its applications
Module 4	• Types of forces (tension, normal, weight, …) • Equilibrium of concurrent forces • FBD
Module 5	• Friction • Coefficient of friction • Static friction • Kinetic friction • Rolling friction • Role of friction in daily life
Module 6	• Dynamics of circular motion • Centripetal force • Banking of roads
Module 7	• Using laws of motion to solve problems in daily life

MODULE 6

3. WORDS YOU MUST KNOW

- Rest: A body is said to be at rest if it does not change its position with time with respect to its surroundings.
- Motion: A body is said to be in motion if it changes its position with time with respect to its surroundings.
- Velocity: The time rate of change of displacement is called velocity.

- Uniform motion: When a particle has equal displacements, in equal intervals of time, (howsoever small this time interval may be) it is said to have a uniform motion. The acceleration for a particle in uniform motion would be zero.

- Momentum (**p**): An indicator of the impact capacity of a moving body. We have

$$\mathbf{p} = m\mathbf{v}$$

- Acceleration: Time rate of change of velocity of a particle, equals its acceleration.

- Vector: A physical quantity that needs both a magnitude and a direction for its specification.
- Vector Algebra: The branch of mathematics that deals with computations involving addition, subtraction, and multiplication of vectors.
- Force: A body will continue in its state of rest, or uniform motion until and unless it is acted upon by an external unbalanced force.

- Inertia: An inherent property of all objects; an object continues in its state of rest or uniform motion unless and until a non-zero external force acts on it.

- Impulse: Rate of change of momentum.

- Static friction: The force of friction which comes into play between the surfaces of two bodies before the body actually starts moving is called static friction.

- Kinetic friction: The force of friction acting between the two surfaces, when one surface is in steady motion over the other surface is called kinetic friction.

- Centripetal force: A force on a body moving in circle acting towards the centre of the circle.

- Angular velocity: A vector quantity describing the speed in terms of angular displacement of an object in circular motion and the direction is perpendicular to the plane of its circular motion. It is given by: v = ωr.

4. INTRODUCTION

Have you ever wondered why the **tyres of vehicles are treaded**? The design on rubber tyre is called **treading**. When we run any vehicle on the road, friction must be influencing the tyres. The wear and tear of tyres as we run our vehicle is commonly known to us. In this unit, we are going to study a way by which vehicles are supported by the forces while negotiating a curved path, like going on rounder's taking a turn etc. you must have also seen a cyclist bend when going on a curve. Banking of roads is a necessary requirement for built in safety, to vehicles moving on a curved road.

CIRCULAR MOTION

We have learnt in the previous modules that acceleration of a body moving in a circle of radius R with uniform speed v is v^2/R directed towards the centre.

According to the second law, the **force f_c providing this acceleration is:**

$$f_c = \frac{mv^2}{R} \quad \ldots\ldots \qquad (1)$$

Where, m is the mass of the body.

This force directed forwards the centre is called the centripetal force.

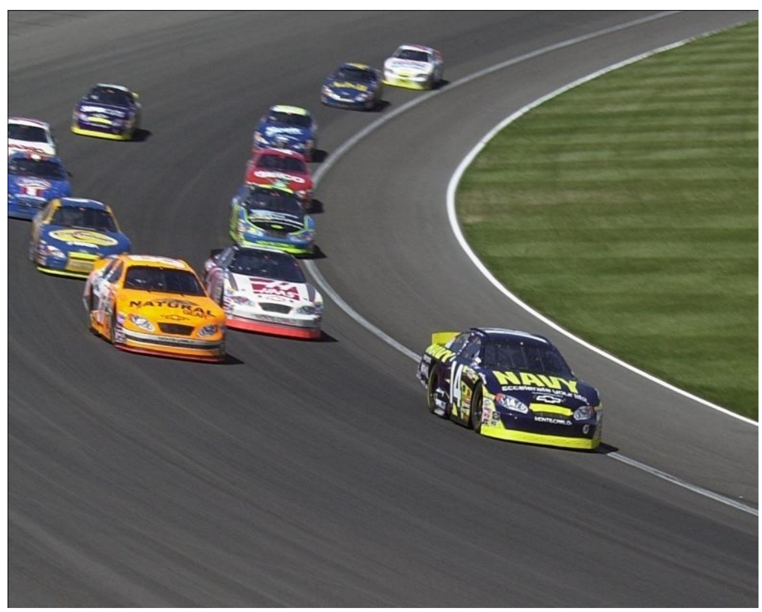
For a stone whirled in a circle by a string, the centripetal force is provided by the tension in the string.

The centripetal force for motion of a planet around the sun is the gravitational force on the planet due to the sun.

For a car taking a circular turn on a horizontal road, the centripetal force is the force of friction.

https://en.wikipedia.org/wiki/File:UK_Roundabout_8_Cars.gif

Open the link to see how vehicles go on the roundabout, because the path is a circle or a part of the circle, centripetal acceleration and hence centripetal force should be present.



The circular motion of a car on a flat and banked road gives interesting application of the laws of motion.

CENTRIPETAL FORCE:

A force that causes motion in a curved path is called a centripetal force.

Uniform circular motion is an example of centripetal force in action. This is the case of body moving with constant speed in a circular path

It can be seen in the orbit of satellites around the earth, a roller coaster looping the loop, or in a bucket swirled around the body.
In previous modules, we learned that any change in a velocity is acceleration. As the object moves through the circular path it is constantly changing direction, and therefore accelerating—causing constant force to be acting on the object. This centripetal force acts toward the centre of circular path. Because the object is moving perpendicular to the force, the path followed by the object is a circular one. It is this force that keeps a body in circular path continuously.

As an object travels around a circular path at a constant speed, **it experiences a centripetal force accelerating it toward the center.**

The equation for centripetal force is as follows:

$$\mathbf{F = \frac{mv^2}{E}}$$

Where

F_c is centripetal force,

m is mass,

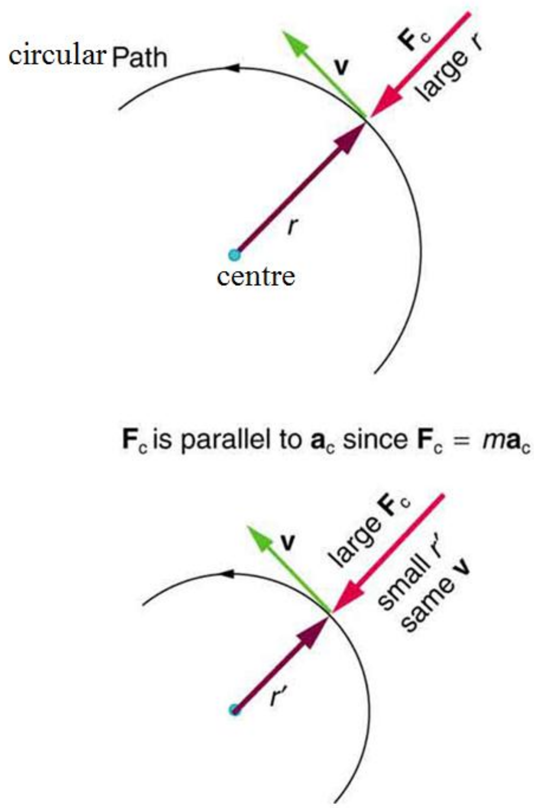
v is speed and

R is the radius of the circular path of motion

Centripetal force can also be expressed in terms of angular velocity. The equation for centripetal force using angular velocity is:

$v = \omega r$

Therefore, $F_c = m R \omega^2$



The following link is about centripetal force

https://youtu.be/hZbxIbVbv7Q

5. BANKING OF ROADS TAKES NORMAL FORCE AND FRICTION INTO ACCOUNT TO PRODUCE CENTRIPETAL FORCE:

BANKING OF ROAD

On a well-designed clean roundabout have you ever noticed the flow of water in case it rains?

The water rushes towards the center of the circular round about, this suggests that the road is inclined towards the center all along the circumference. Picture 1

The raising of one side of road surface with respect to the other in a gradual manner, creating a slope is called banking.

The design of raising the outer edge of the curved road above the inner edge is called banking of roads and the angle through which it is raised is called angle of banking.

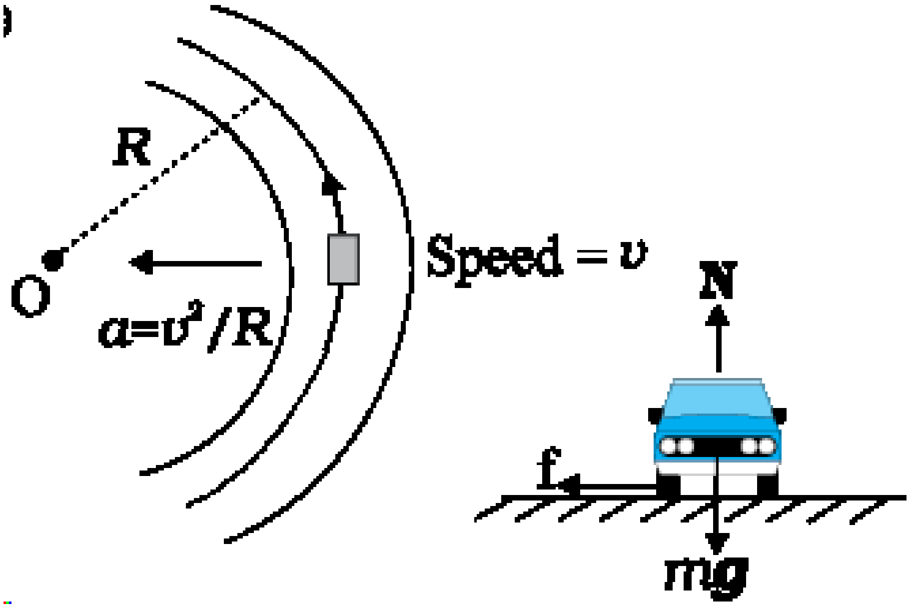
On a cycle race tracking a velodrome, this is visibly evident as in the picture 2

Picture 1

Picture 2

In order to understand why governments or road makers bank a road let us study the following schematic diagrams

MOTION OF A CAR ON A LEVEL ROAD



Circular motion of a car on a level road

When a car is moving on a level road, **three forces act on the car are:**

(i) **The weight of the car, mg**

(ii) Normal force, N

(iii) Frictional force, f (along the surface of the road, towards the centre of the circular turn)

$N - mg = 0$

Or $N = mg$

As there, is no motion in the vertical direction.

The centripetal force required for circular motion is along the surface of the road, and is provided by the component of the contact force between road and the car tyres along the surface.

This, by definition, is the frictional force.

Note that it is the static friction that provides the centripetal acceleration. Static friction opposes the impending motion of the car, moving away from the circular path.

So, we get

$$f_s \geq \frac{mv^2}{R}$$

$$\mu_s N \geq \frac{mv^2}{R}$$

where μ_s is the coefficient of static friction and R is the radius of the circular track along which car travels.

Using the above equations, we get:

$$\mathbf{v^2 \leq \frac{\mu_s RN}{m} = \mu_s Rg}$$

$$\mathbf{f \leq \mu_s N = \frac{mv^2}{R}} \qquad [\because\ N = mg]$$

$$\mathbf{v_{max} = \sqrt{\mu_s Rg}}$$

which is **independent of the mass of the car.**

This shows that for a given value of μ_s and R, there is a maximum speed of circular motion of the car possible, namely $v_{max} = \sqrt{\mu_s Rg}$

EXAMPLE

Find the maximum speed with which a car can turn on a bend without skidding, if radius of bend is 20 m and coefficient of friction between road and tyre is 0.4.

SOLUTION:

R = 20 m μ_s = 0.4 v =?

$v_{max} = \sqrt{\mu_s R g} = \sqrt{20 \text{ x } 0.4 \text{ x } 9.8}$

= 28 m/s

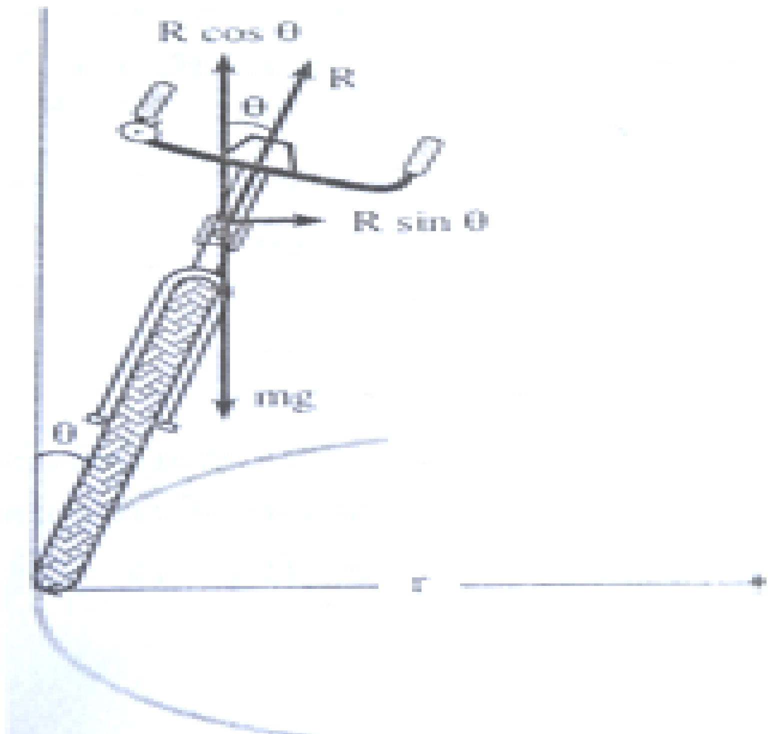
LET US UNDERSTAND

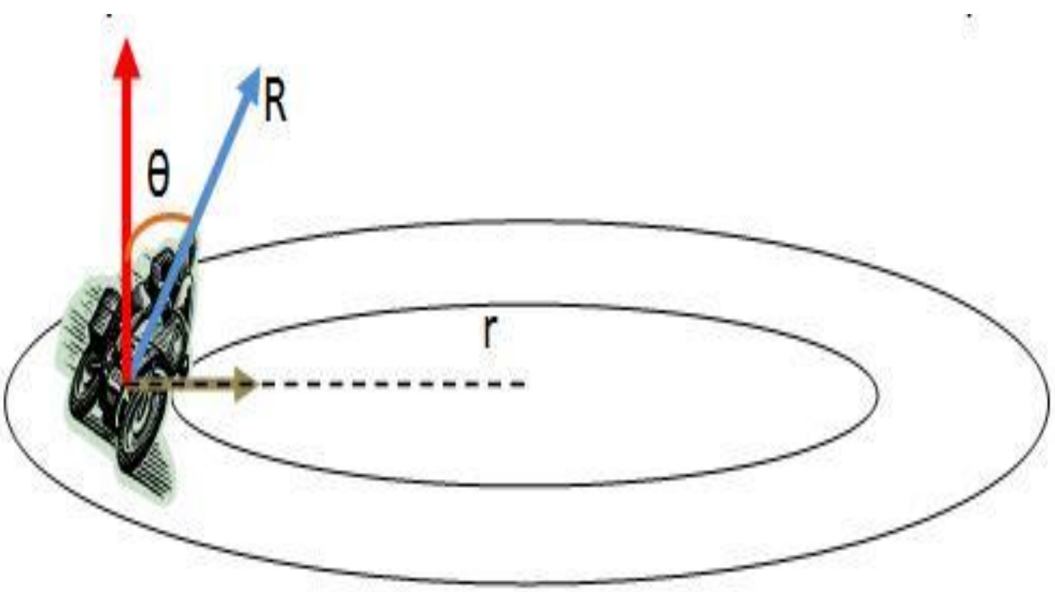
BENDING BY A CYCLIST ON A LEVEL ROAD

We have generally observed that a cyclist bends towards the center in order to move along a circular path.

The cyclist increases speed without skidding by leaning towards the centre of circular path.

The sole objective of bending here is to change the direction and magnitude of normal force such that horizontal component of the normal force provides for the centripetal force, whereas vertical component balances the "cycle and cyclist" body system.





The horizontal component of normal force meets the requirement of centripetal force.

$R \cos \theta = m g$

R sin θ $= \frac{mv^2}{r}$

Taking ratio,

$\Rightarrow tan\ \theta = \frac{v^2}{R\ g}$

$\Rightarrow$ v = √(r g tan θ)

EXAMPLE:

A cyclist speeding at 1.25 m/s on a level road takes a sharp turn of radius 3 m without reducing the speed. The coefficient of friction between the road and tyre is 0.1. Will he slip while taking the turn?

SOLUTION:

Maximum speed for not slipping is

v = √(r g tanθ)

= √0.1 X 3 X 9.8

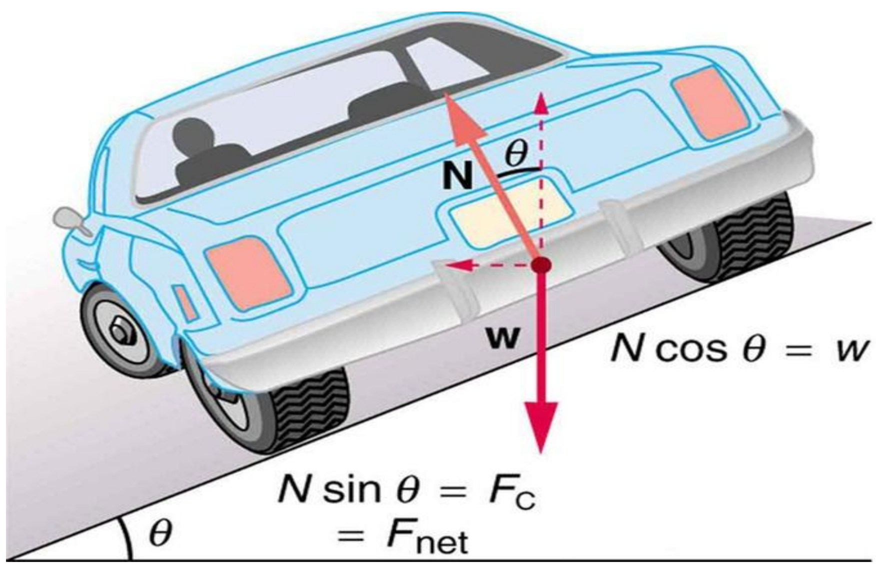
= 1.71 m/s

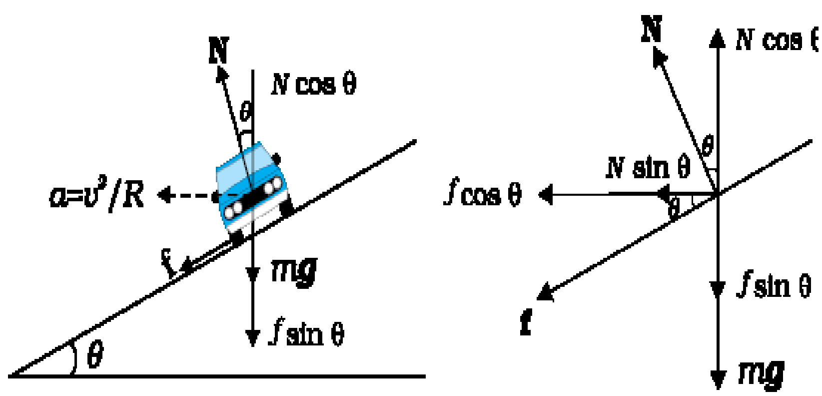
As he is moving with 1.21 m/s, he will not slip.

For ideal banking, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively. In the cases in which forces are not parallel, it is more convenient to consider components along perpendicular axes—in this case, the vertical and horizontal directions.

WHY IS BANKING OF ROADS IMPORTANT?

The design of raising the outer edge of the curved road above the inner edge is called banking of roads and the angle through which it is raised is called angle of banking.





Above is a free body diagram for a car on a **frictionless banked curve**.

The only two external forces acting on the car are its weight and the normal force of the road. The small force of friction acts along the surface

These two forces must add to give a net external force that is horizontal toward the centre of circular track and has magnitude .

Only the normal force has a horizontal component, and so this must be equal to the centripetal force — that is:

$$N \sin \theta = \frac{mv^2}{R}$$

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction.

From the figure, we see that the vertical component of the normal force is N cos θ, and the only other vertical force is the car's weight. These must be equal in magnitude, Thus,

$$N \cos \Theta = Mg$$

Dividing the above equations yields:

$$tan\ \theta = \frac{v^2}{R\ g}$$

Taking the inverse tangent gives:

$$\theta = \frac{tan^{-1} v^2}{R\ g}$$

For an ideally banked curve with negligible friction,

⇒ v = √(r g tanθ)

This expression can be understood by considering how θ depends on v and R.

A large θ will be obtained for a large v and a small R.

That is, **roads must be steeply banked for high speeds and sharp curves.** Friction helps, because it allows you to take the curve at greater or lower speed than if the curve is frictionless (or with negligible friction). Note **that θ does not depend on the mass of the vehicle**.

We can reduce the contribution of friction to the circular motion of the car if the road is banked.

Since there is no acceleration along the vertical direction, the net force along this direction must be zero. Hence,

N cos θ = mg + f sin θ (1)

The centripetal force is provided by the horizontal components of N and f.

N sin θ + f cos θ = $\frac{mv^2}{R}$ (2)

But f ≤ $\mu_s N$

Thus to obtain v_{max}, we put

Then, Eq. (1) and eq.(2) become

N cos θ = mg + $\mu_s N$ sin θ (3)

N sin θ + $\mu_s N$ cos θ = $\frac{mv^2}{R}$ (4)

From Eq. (3), we obtain

$$N = \frac{mg}{\cos\theta - \mu_s \sin\theta}$$

Substituting value of N in Eq. (4), we get

$$\frac{mg(\sin\theta + \mu_s\cos\theta)}{\cos\theta - \mu_s\sin\theta} = \frac{m{v_{max}}^2}{R}$$

$$v_{max} = \left(Rg\frac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}\right)^{\frac{1}{2}}$$

Comparing this with Equation:

$$v_{max} = \sqrt{\mu_s R g}$$

We see that maximum possible speed of a car on a banked road is greater than that on a flat road.

For $\mu_s = 0$

$$v_0 = v_{max} = (R\ g \tan\theta\ )^{1/2}$$

We had also seen that at this optimum speed, v_0, frictional force is not needed at all to provide the necessary centripetal force.

Driving at this speed on a banked road will cause little wear and tear of the tyres.

The same equation also tells you that for $v < v_o$, frictional force will be up the slope and that a car can be parked only if $\tan\theta \le \mu_s$.

The following video is for banking of roads.

https://youtu.be/jvygNOsbcic

https://en.wikipedia.org/wiki/Banked_turn (for banking of roads)

Now you will appreciate that the speed limit on curved roads is fixed for two reasons

i) To avoid accidents as drivers may lose control over high speed vehicles

ii) The tyres would not wear and tear as friction is not being used to provide centripetal force

EXAMPLE

A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The co-efficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn?

SOLUTION:

On an unbanked road, frictional force alone can provide the centripetal force needed to keep the cyclist moving on a circular turn without slipping. If the speed is too large, or if the turn is too sharp (i.e. of too small a radius) or both, the frictional force is not sufficient to provide the necessary centripetal force, and the cyclist slips. The condition for the cyclist not to slip is given by:

$v^2 \leq \mu_s R g$

Now, R = 3 m, g = 9.8 m s^{-2}, $\mu_s = 0.1$.

That is, $\mu_s R g = 2.94\ m^2 s^{-2}$. v = 18 km/h = 5 m s^{-1}; i.e., $v^2 = 25\ m^2 s^{-2}$.

The condition is not obeyed. The cyclist will slip while taking the circular turn.

EXAMPLE:

A circular race-car track of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2, what is the:

(a) Optimum speed of the race-car to avoid wear and tear on its tyres, and

(b) Maximum permissible speed to avoid slipping?

SOLUTION:

a) On a banked road, the horizontal component of the normal force and the frictional force contribute to provide centripetal force to keep the car moving on a circular turn without slipping. At the optimum speed, the normal force's component is enough to provide the needed centripetal force, and the frictional force is not needed. The optimum speed v_o is given by):

$$v_0 = (R\ g \tan\theta\ )^{1/2}$$

Here R = 300 m, θ = 15°, g = 9.8 m s^{-2}; we have

v_0 = 28.1 m s^{-1}.

b) The maximum permissible speed v_{max} is given by:

$$v_{max} = \left(Rg\,\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)^{\frac{1}{2}}$$

$$= 38.1 \text{ m/s}$$

EXAMPLE:

An aircraft hovers over a city awaiting clearance to land. The aircraft circles with its wings banked at an angle $\tan^{-1}$ (0.2) at a speed of 200 m/s. Find the radius of the loop.

SOLUTION: The aircraft is banked at an angle with horizontal. Since aircraft is executing uniform circular motion, a net force on the aircraft should act normal to its body. The component of this normal force in the radial direction meets the requirement of centripetal force, whereas vertical component balances the weight of aircraft. Thus, this situation is analogous to the banking of road.

$$\Rightarrow \qquad \tan\theta = \frac{v^2}{rg}$$

$$r = \frac{v^2}{g\tan\theta}$$

$$\Rightarrow \qquad r = \frac{200^2}{10\times 0.2}$$

$$= 20000m = 20\ km$$

6. SUMMARY

- A force that causes motion in a curved path is called a centripetal force
- The equation for centripetal force is as follows:

  $F_c = mv^2/R$

- The maximum speed of circular motion of the car on a level road is

$$v_{max} = \sqrt{\mu_s R g}$$

- Maximum speed of the cyclist taking a circular turn for not slipping is

  $v = \sqrt{(R\, g \tan\theta)}$

The maximum possible speed of a car on a banked road is:

$v_{max} = \left(Rg\frac{\mu_s + \tan\theta}{1-\mu_s \tan\theta}\right)^{\frac{1}{2}}$ which is greater than that on a flat road.